System-Level Optimization of Longitudinal Acceleration of Autonomous Vehicles in Mixed Traffic

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***Abstract*— Mixed traffic scenarios present challenges to autonomous vehicles due to the high degree of randomness introduced by human drivers combined with their larger reaction times and perception errors. In this paper we address those challenges on a longitudinal control level by designing optimal car-following models which aim to maximise simultaneously the vehicle population's speed, efficiency, comfort, and safety. We use the agent-based simulation mixed traffic tool BEHAVE to design a scenario covering all driving phases and formalize the four different objective functions to be optimized. We take on a multi-objective optimization approach in order to analyse the trade-offs that occur between the chosen traffic metrics. Furthermore, we design a methodology to scalarize the multi-objective problem and find a single optimal well-balanced parameter set maximizing the formulated objective functions. The optimized model is able to gain significant performance increase in terms of efficiency, comfort and safety, while giving away a significantly smaller percentage of average speed.**

I. INTRODUCTION

Inevitably our roads will be shared between autonomous vehicles (AVs) and human drivers. With the increase of self-driving cars' technology readiness level, mixed traffic scenarios involving both humans and AVs present bigger interest to researchers, policy makers, and original equipment manufacturers (OEMs). It is widely accepted that AVs will have beneficial effects on traffic parameters such as efficiency and safety in a purely autonomous environment, however, mixed traffic conditions pose challenges that might hinder the magnitude of those improvements.

Autonomous vehicles can perform well together in platoons due to their predictable behaviour, small reaction times, and communication capabilities. When an AV is behind a human-driven vehicle, however, it is capable of making a limited amount of assumptions about the human's future behaviour and has to infer the driver's intentions instead of receiving them over a communication channel. This calls for an overly cautious driving style that can reduce the overall traffic system performance. The magnitude of this performance deterioration naturally attracts a great interest, however, a more fundamental question that we try to address in this work is what metrics define the performance of the traffic system.

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While speed and efficiency (in terms of energy needed per unit of distance) are among the most discussed metrics, comfort and safety are often omitted or studied in isolation. In order to fill this gap, we study the effects of speed, efficiency, comfort, and safety together as we believe they are all equally important factors that need to be considered.

Currently there are not enough AVs on the roads to perform real-life experiments to estimate those metrics. Therefore, traffic simulation is a key tool that enables researchers to study mixed traffic's implications on traffic conditions. While macroscopic city-scale mixed traffic studies exist [1], since we are interested in studying the interaction of AVs and humans, we require a microscopic agent-based simulation.

In this paper we make use of the BEHAVE tool which is designed specifically to study agent-based interactions in mixed traffic conditions on a multiple lane highway [2]. The tool allows the user to specify different agent groups governed by different driver behaviour models which is precisely what is needed to study mixed traffic scenarios.

There are two main components describing vehicle motion; longitudinal and lateral movement. In this paper, we will focus on the longitudinal motion in order to reduce the parameter space and the complexity of our analysis. In agent-based simulations, the longitudinal motion of vehicles is governed by a car-following model, which can be perceived as a function that takes inputs from the environment of the vehicle and returns the acceleration that it would like to apply.

The goal of this paper is to study how existing models for longitudinal acceleration of AVs perform in a mixed traffic context and how they can be improved to maximize traffic performance metrics such as speed, efficiency, comfort, and safety. More specifically, we choose two prominent car-following models and try to find optimal parameter sets for them. As we have multiple metrics to maximize, we take a multi-objective optimization approach that allows us to explore the trade-offs between the considered objectives. Using the results obtained from the multi-objective optimization we combine the objectives, which allows us to come up with a single parametrization of the two initially chosen models that balances well between the four different objectives.

The contributions of this paper can be summarized as:

- Mathematical formulation of objective functions for speed, efficiency, comfort, and safety in mixed traffic
- Comparison of Pareto fronts of optimized AV longitudinal control models
- A methodology to choose a single parametrization of a control model

II. Related Work

There are two main types of car-following models: stimulus-response models and collision avoidance models. The stimulus-response models capture the behaviour of the "following" vehicle related to some stimulus coming from the "leading" vehicle. One example of a stimulus-response model is the optimal velocity model (OVM) proposed initially by Bando et al. [3]. It relies on differential equations describing the velocity and acceleration of the agent, depending on the velocity and position of the vehicle ahead and a sensitivity parameter denoting the speed of the response. It has been reported, however, that the model can encounter unrealistic acceleration and deceleration values due to the fact that it reacts to insignificant stimuli from the vehicle ahead.

This problem is addressed in the generalized force model (GFM) of Helbing and Tilch [4] by addition of a term which takes into consideration the difference of velocities between the vehicles. This, however, introduces an unrealistic behaviour at close proximity when the lead vehicle is travelling faster than the follower. The full velocity difference model (FVDM) postulated in by Jiang et al. [5] is an extension of the GFM, which also takes such situations into consideration but makes no allowance of the effect of the inter-car spacing independently of the relative velocity [6]. Furthermore, it must be noted that the GFM, OVM and FVDM require very small simulation time-steps in order to produce realistic acceleration patterns, which makes them computationally inefficient [7]. Additionally, the responsive nature of these models results in agents constantly reacting to non-essential stimuli thus leading to unrealistic behaviour.

A widely used example of a collision avoidance car-following model is Gipps' model described by Gipps in [8]. It utilizes the concept of maintaining a safe speed in order to avoid collisions. As a consequence of the safe speed and safe distance conditions Gipps' model typically underestimates the capacity of the system as drivers keep too large distances between themselves since they are always prepared for the worst-case scenario.

The Intelligent Driver Model (IDM) described by Treiber et al. in [9], although considered a stimulus-response model, incorporates the concept of safe driving similarly to Gipps' model. It can be thought of as a hybrid model unifying the two categories described above.

There is a safe distance, however, approaching slower vehicles is smoother, as there is no fixed deceleration but rather a gradually increasing deceleration until the comfortable deceleration value is reached. The IDM offers smooth transitions between the different driving modes: acceleration, deceleration, car-following, which is manifested in a more realistic acceleration profile and is formulated in Equation 1.

$$a_i = a_0 \left(1 - \left(\frac{v}{v_0} \right)^{\beta} - \left(\frac{s_0 + vT_0 + \frac{v(v-v_l)}{2\sqrt{a_0 b_0}}}{s} \right)^2 \right) \tag{1}$$

The IDM has the following parameters: preferred speed v_0, preferred acceleration a_0, preferred deceleration b_0, minimum gap s_0, time headway T_0 (time it would take for the vehicle to reach the vehicle ahead's tail position), free road term factor β. The inputs to the model are the vehicle's velocity v, velocity of vehicle ahead v_l, and distance to vehicle ahead s. The enhanced IDM (E-IDM) suggested in [10] has an additional parameter called coolness c which allows for a less reactive behaviour of the model while keeping it collision free thus improving traffic flow. The acceleration in the enhanced IDM is computed as follows:

$$a_e = \begin{cases} a_i & if \quad a_i \geq a_c \\ (1-c)\, a_i + & \\ c\left(a_c + b_0 \tanh\left(\frac{a_i - a_c}{b_0} \right) \right) & otherwise \end{cases} \tag{2}$$

where a_c is called the constant acceleration heuristic and is formally defined as:

$$a_c = \begin{cases} \frac{v^2 \tilde{a}_l}{v_l^2 - 2s\tilde{a}_l} & if \quad v_l(v - v_l) \leq -2s\tilde{a}_l \\ \tilde{a}_l - \frac{(v-v_l)^2 \Theta(v - v_l)}{2s} & otherwise \end{cases} \tag{3}$$

where $\tilde{a}_l = min(a_l, a)$ is called the effective acceleration and Θ is the Heaviside step function.

The afore-mentioned suitability of the IDM as a model that accurately represents traffic dynamics make it one of the fundamental building blocks used in this paper. More specifically, the E-IDM is used as one of the two candidates for AV control in the mixed traffic scenario. In [10] it is shown that vehicles employing the algorithm improve the throughput of the system, however, comfort, safety, and efficiency are not analysed.

The second model that we will consider is a stimuli algorithm that is typically used for ACC and that is extended into the CACC version that AVs utilize in platoons. Since we create the mixed traffic conditions such that there are no platoons, CACC is not a suitable option, however, the control logic of ACC described in [11] looks promising. It consists of a controller that follows the speed of the vehicle in front with a minimal delay with a control parameter α, and keeps a predefined time headway T_0. The model is formulated as in Equation 4.

$$a_{acc} = -\frac{1}{T_0} \left(v - v_{\mathrm{l}} + \alpha \left(T_0 v - s \right) \right) \tag{4}$$

In order to model human drivers in our simulations, we use the Human Driver Model (HDM), an extension of the IDM described in [12]. This extension consists of 4 main parts: addition of reaction time, perception errors, temporal anticipation of other drivers, which aims at counteracting the delay in reaction time, and spatial anticipation of more than one vehicle ahead.

The reaction time T_r is implemented by evaluating the right-hand side of Equation 1 at time $t - T_r$:

$$a_i = \left| a_0 \left(1 - \left(\frac{v}{v_0} \right)^{\beta} - \left(\frac{s_0 + vT_0 + \frac{v(v-v_l)}{2\sqrt{a_0 b_0}}}{s} \right)^2 \right) \right|_{t-T_r} \tag{5}$$

The perception errors are modelled using a Wiener process applied on the inter-vehicle distance and approaching speed $\Delta v = v - v_l$:

$$s^{est} = s \exp(V_s w_s t) \tag{6}$$
$$\Delta v^{est} = \Delta v + s r_c w_{\Delta v} \tag{7}$$

where V_s is the variation coefficient for spatial error, r_c inverse TTC error, and w_s and $w_{\Delta v}$ are Wiener processes defined as:

$$w(t + \Delta t) = \exp(-\Delta t/\tau)\, w(t) + \sqrt{\frac{2\Delta t}{\tau}} \eta(t) \tag{8}$$

where τ is the error correlation time and $\eta(t)$ is sampled from a normal distribution.

The temporal anticipation of the driver represents the IDM inputs estimation at time t from their values at $t - T_r$. Those estimations are computed by assuming either constant or no acceleration:

$$\hat{s} = \left| s^{est} - T_r \Delta v^{est} \right|_{t-T_r} \tag{9}$$
$$\hat{v} = \left| v^{est} + T_r a \right|_{t-T_r} \tag{10}$$
$$\hat{\Delta v} = \left| \Delta v^{est} \right|_{t-T_r} \tag{11}$$

The IDM equation is then used with the adjusted inputs. The spatial anticipation of the driver is modelled by including interaction terms for more vehicles ahead. The original IDM formula can be split into a free road (first term in Equation 1) and an interaction term (second term in Equation 1).

$$a_i(s, v, \Delta v) = a^{free}(v) + a^{int}(s, v, \Delta v) \tag{12}$$

The interaction term itself takes the inputs of the model $(s, v, \Delta v)$. When interaction terms for multiple vehicles ahead are added, therefore, the respective new inputs to the interaction term are computed:

$$a_{hdm} = a^{free}(\hat{v}) + \sum_{k=1}^{n_a} a_k^{int}(\hat{s_k}, \hat{v}, \hat{\Delta v_k}) \tag{13}$$

where the term a_k^{int} is the interaction term with the k-th vehicle in front and n_a is the number of anticipated vehicles.

III. Scenario

The scenario we design in order to test out various car-following models aims at capturing all parts of longitudinal driving. Those include starting from a standstill position, acceleration phase, steady velocity driving phase, unexpected braking, and recovery. Since we examine only car-following behaviour we use a straight road with a single lane for our experiments. The vehicles are ordered by alternating human-driven and AVs in order to fully observe the interaction between the two types of vehicles. The human-driven vehicles use the HDM model described in the previous section with the following parameters: $a_0 = 2$, $b_0 = 2$, $T_0 = 1.5$, $T' = 1.5$, $V_s = 0.05$, $r_c = 0.01$, $\tau = 20$, based on [12]. The models used by the autonomous vehicles are altered depending on the experiment that is being preformed (either ACC or E-IDM). All vehicles start from velocity 0 positioned one vehicle length apart. The time step of the simulation is set to $\Delta t = 0.1$ seconds. One minute after the start of a simulation run (episode), the leading vehicle brakes with constant deceleration b for a time duration of d seconds, after which the simulation continues running for one more minute. One experiment consists of 5 episodes with increasing deceleration and braking duration in order to evaluate the overall performance of the population for a variety of braking situations. The specific values of b and d chosen for the episodes are: $b = \{-2, -4, -6, -8, -10\}$ m/s^2 and $d = \{1, 2, 3, 4, 5\}$ seconds. Figure 1 shows the acceleration profile of a small vehicle population with AVs that utilize the ACC model.

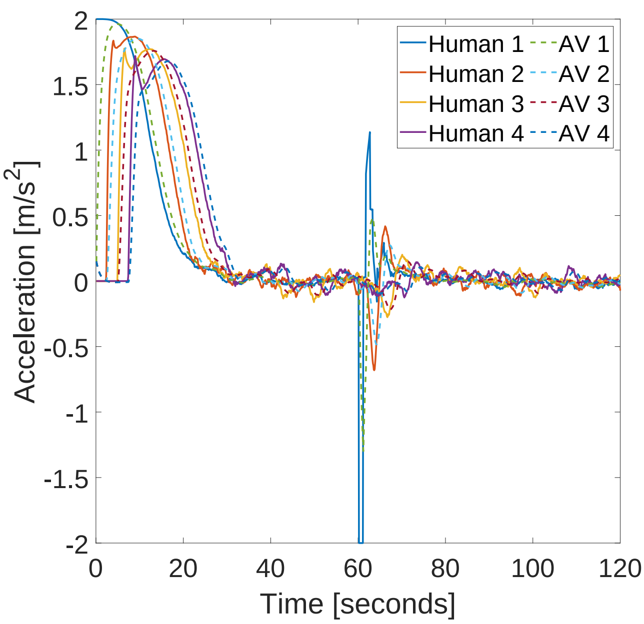

Fig. 1: Acceleration profile of an 8 vehicle population where AVs use the ACC model.

For the experiments run throughout the paper we use a population of 50 vehicles. Since the human model introduces randomness to the experiment, every experiment (5 episodes) is run 10 times and the results are averaged.

IV. Optimization Objectives

The four metrics that we have determined relevant for AVs are speed, efficiency, comfort, and safety. Ideally, a car-following model will increase all of them, however, typically there are trade-offs. The following subsections discuss in detail how we have measured those traffic parameters. The notation used to describe the objectives can be found in Table I.

Symbol	Description
N	number of agents
K	number of time steps
$a^{i,k}$	acceleration of i-th agent at time step k
$v^{i,k}$	velocity of i-th agent at time step k
$p^{i,k}$	position of i-th agent at time step k
Δt	time step duration
ρ	air density
c_w	drag coefficient
M	total number of episodes in an experiment run
A	frontal area of vehicle
ϕ	rolling friction coefficient
m	mass of vehicle
g	gravitational acceleration
λ	weight adjustment factor
η_{rb}	efficiency of recuperative braking
ψ	time-to-collision critical threshold

TABLE I: Notation

A. Speed

The average speed $\hat{V}_j$ of the population during episode j is computed by adding up the travelled distances over the episode of all cars and dividing this by the total travel time of the vehicles. The overall average speed metric $\hat{V}$ is computed as a weighted sum of the episodes. The episode with the highest weight is the one with the smallest braking intensity as it is more frequent in real life situations.

$$\bar{V}_j = \frac{\sum_{i=2}^{N} p_{i,K}}{(N-1)\Delta t K} \tag{14}$$

$$\bar{V} = \sum_{j=1}^{M} \alpha_j \hat{V}_j \tag{15}$$

Where $\alpha_j = \frac{1/2^j}{\sum_{j=1}^{M} 1/2^j}$. The first vehicle is excluded from this computation because in all scenarios its average speed remains the same.

B. Efficiency

The efficiency of the vehicle population throughout the episodes is determined by computing the distance that could be driven with one unit of energy. As with the speed, the episode efficiencies are combined using a weighted sum to produce an overall efficiency. The energy consumption of a vehicle is computed by using a simple model for the power needed or generated by the vehicle's accelerations. There are three terms that are used to arrive at the total power: the air friction P^{air}, rolling friction P^{roll}, and acceleration P^{acc} power terms.

$$P^{\text{air}}_{i,k} = 0.5\rho c_w A v^3_{i,k} \tag{16}$$

$$P^{\text{roll}}_{i,k} = \phi m g v^3_{i,k} \tag{17}$$

$$P^{\text{acc}}_{i,k} = \begin{cases} m(1+\lambda)a_{i,k}v_{i,k}, & \text{if } a \geq 0 \\ m(1+\lambda)a_{i,k}v_{i,k}\eta_{\text{rb}}, & \text{otherwise} \end{cases} \tag{18}$$

$$P_{i,k} = P^{\text{air}}_{i,k} + P^{\text{roll}}_{i,k} + P^{\text{acc}}_{i,k} \tag{19}$$

where η_{rb} is the efficiency of the recuperative braking (in the case of electric AVs) modelled as $\left[e^{\frac{0.0411}{|a_{i,k}|}}\right]^{-1}$ [13]. The total energy consumption E_j for episode j is computed by using the power consumption of every vehicle for every time step multiplied by the time step duration. Similarly to the average speed case the overall efficiency is a weighted sum of the episode efficiencies.

$$E_j = \sum_{i=2}^{N} \sum_{k=1}^{K} P_{i,k} \Delta t \tag{20}$$

$$E = \sum_{j=1}^{M} \alpha_j E_j \tag{21}$$

C. Comfort

The overall passenger comfort C_j for episode j in the population is measured by computing the total jerk that was experienced throughout the episode. We have assumed that both positive and negative changes in acceleration affect discomfort equally. Since we are trying to compute the comfort we negate the sum of jerks, which represents the discomfort.

$$C_j = \sum_{i=2}^{N} \sum_{k=1}^{K-1} -|a_{i,k} - a_{i,k+1}| \tag{22}$$

$$C = \sum_{j=1}^{M} \alpha_j C_j \tag{23}$$

D. Safety

The overall safety S_j of episode j is computed by utilizing one of the most widely known safety surrogate metrics time-to-collision (TTC) [14]. We represent the risk of an accident as the duration for which the TTC between two vehicles is below a certain critical value ψ. Literature indicates that this threshold for rural roads similar to our scenario is typically set to $\psi = 3$ seconds [15], [16], [17]. As we would like to represent the safety with our metric we negate the overall duration for which the TTC is under the critical threshold.

$$S_j = \sum_{k=1}^{K} \sum_{i=2}^{N} -\Theta(\psi - TTC_{i,k})\Delta t \tag{24}$$

$$S = \sum_{j=1}^{M} \alpha_j S_j \tag{25}$$

where Θ is the Heaviside step function.

V. RESULTS AND DISCUSSION

Because we have multiple objective functions, we need to employ a multi-objective optimization approach.

The optimization problem can be formulated as follows:

$$\arg\min_{\omega_{\text{CF}}}(-\bar{V}, -E, -C, -S) \tag{26}$$

where ω_{CF} is the set of parameters for car-following model CF.

As discussed in Section II we have chosen two different models to evaluate in this paper. The first one is the enhanced IDM (E-IDM) and the second one is the ACC as described in Equation 4. We solve the optimization problem by using the NSGA-II genetic algorithm [18]. The model parameters ω_{CF} and their support for the two models are as follows:

- **Enhanced IDM:** Preferred acceleration $a_0 \in [0.5, 4]$, preferred deceleration $b_0 \in [0.5, 4]$ time headway $T_0 \in [0.5, 4]$ and free road term factor $\beta \in [0.5, 4]$.
- **ACC:** time headway $T_0 \in [0.5, 4]$ and control parameter $\alpha \in [0.01, 1]$

The only constraints of the optimization problem are the defined supports of the model parameters ω_{CF}. Those upper and lower parameter bounds were chosen in order to provide a reasonably large exploration space while keeping the parameter values realistic, and thus not too far away from the parameters human drivers exhibit. In the case of E-IDM, the inter-vehicle distance s_0 is not being studied since initial tests showed insignificant effects on the objective functions. Furthermore, the preferred speed v_0 is not included in the parameter list since it controls the same part of the function as the free road term factor.

A. Multi-objective optimization of ACC and EIDM

The NSGA-II algorithm is run with parameters for a maximum of 100 iterations with population size 50. The mutation rate is set to 0.01 and the mutation percentage is 0.4. The optimization process produces a four dimensional Pareto front. Figure 2 shows the four most interesting two dimensional projections of the front. Since the main trade-off is between the average speed and the rest of the objectives, the front is best observed on those projections.

It can be observed the E-IDM and ACC form a continuous front with the ACC taking the high velocity part of the front while the E-IDM is able to reach more efficient and comfortable solutions. Both models are able to produce solutions with the maximum safety value (0), however, the E-IDM has a solution that has slightly higher average speed. The safety-comfort projection clearly demonstrates that there is no trade-off between those two objectives as the maximum safety and comfort values are actually reachable by a single parametrization of the E-IDM model.

None of the fronts shows a clear domination of one model over the other for the regions of the front where the non-dominated solutions coexist. The fact that none of the models dominates the other in the shared parts of the front could imply that both models are derived from the same master

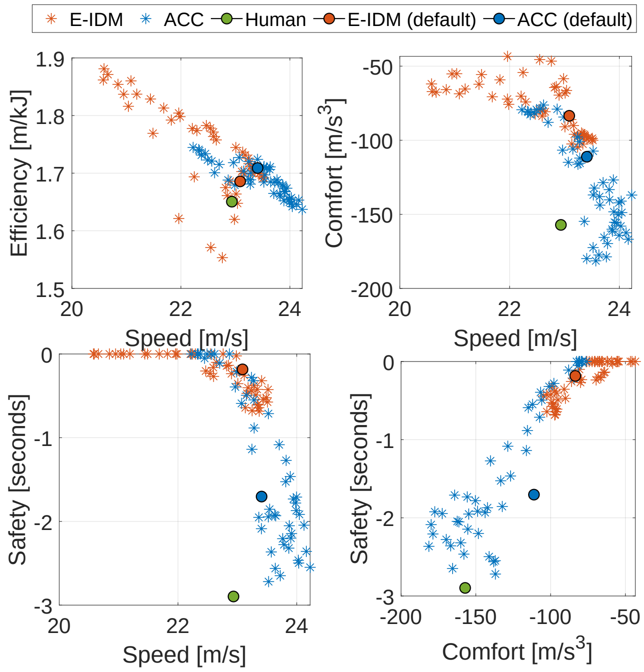

Fig. 2: Projections of Pareto fronts (*) and default parameter solutions (circles) for purely human population (green) and mixed traffic scenarios with AVs governed by E-IDM (orange) or ACC(blue) models.

model similarly to what was demonstrated in [19] for the IDM and the OVM models.

The reference point of the E-IDM with parameters $a_0 = 2$, $b_0 = 2$, $T_0 = 1.5$, and $\beta = 4$ resides well within the front for comfort and safety, which means that the parametrization remains optimal for those objectives. It is, however, an internal point of the efficiency front which means that there exist better parametrizations which dominate it. The reference point of the ACC model is dominated on the safety front and resides within the other fronts, which means that there are better parametrizations, which allow safer operations of ACC controlled vehicles.

The human model reference point is dominated by all solutions on the Pareto front. This is an expected finding, which reconfirms the proposition that intelligently controlled autonomous vehicles can simultaneously improve the speed, efficiency, comfort, and safety of the traffic system.

B. Scalarization of Optimization Problem and Comparison of Balanced Solutions

The results of the multi-objective optimization have provided us with bounds for the chosen objective functions. These bounds help us define what are realistically the minimum and maximum values of the objectives functions where optimal behaviour in a multi-objective sense can be observed. We can use those bounds to normalize the objective functions and then combine them into a single objective, which will

	Default E-IDM	Default ACC	Balanced E-IDM	Balanced ACC
Improvement of Average Speed [%]	0.39	3.21	-2.47	-0.22
Improvement of Efficiency [%]	1.76	0.58	6.96	2.38
Improvement of Comfort [%]	44.29	21.63	50.82	36.08
Improvement of Safety [%]	93.82	59.94	100	95.47

TABLE II: Improvement of traffic system performance of various AV driving models over human-driver only scenario.

allow us to find a single balanced solution. Once minimum and maximum values from all non-dominated solutions of both models for all of the objectives are collected the single objective for the performance of the traffic system P can be formulated as:

$$\begin{aligned} P = \alpha_1 \left(\frac{\bar{V} - \bar{V}_{\text{min}}}{\bar{V}_{\text{max}} - \bar{V}_{\text{min}}} \right) + \alpha_2 \left(\frac{E - E_{\text{min}}}{E_{\text{max}} - E_{\text{min}}} \right) \\ + \alpha_3 \left(\frac{C - C_{\text{min}}}{C_{\text{max}} - C_{\text{min}}} \right) + \alpha_4 \left(\frac{S - S_{\text{min}}}{S_{\text{max}} - S_{\text{min}}} \right) \end{aligned} \tag{27}$$

The weights α_{1-4} can be set to be equal or can be used to prioritize certain objectives over others or set manually to values depicting the priorities of the practitioner that is using this methodology. In this paper we choose to have three levels of priority. Highest priority is given to the speed and safety objectives ($\alpha_1 = \alpha_4 = 3$), medium priority is given to efficiency ($\alpha_2 = 2$), and the lowest priority is given to comfort ($\alpha_3 = 1$).

The new optimization problem is formulated as $\underset{\omega_{\text{CF}}}{\arg\min}\, P$. We utilize the simplex algorithm in order to find the optimal solutions for the parameters of both the E-IDM and the ACC. The results of the solutions and their comparison to the reference points and the human driver model can be found in Table II.

It can be observed that the balanced solutions in both cases "borrow" from the speed objective in order to improve on the efficiency, comfort, and safety metrics. The increase in efficiency by the E-IDM balanced model parametrization is more than twice as big as the one of the balanced ACC model. Furthermore, the comfort is increased by a factor of 1.5 more and safety reaches the perfect score, which means that no car in any of the episodes has crossed the 3 seconds TTC critical threshold. The ACC balanced model parametrization is, however, producing a higher speed metric, by only sacrificing 0.22% of the average speed value, while the E-IDM is losing 2.47%.

An overarching observation from our results is that the ACC model parametrizations offer higher average velocities and lower comfort, efficiency, and safety than the E-IDM. Smooth transitions between driving phases and relaxed reactions to changing traffic conditions, typical for the E-IDM favour comfort, efficiency, and safety, but also lead to slower reaction to changing conditions. In some cases this would mean missed opportunities for steeper, and therefore faster, acceleration

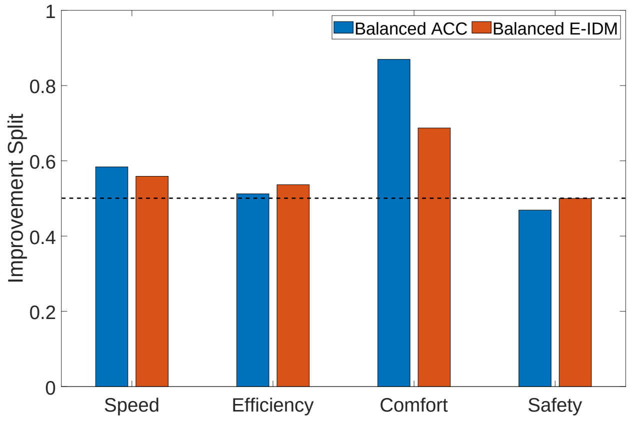

Fig. 3: Proportion of improvement in traffic performance objectives received by AVs for the two balanced models.

phases. One key detail in our scenario description is that we include the acceleration phase of driving in our experiments rather than initiating our vehicles at cruising speed as it is usually done. This more complete evaluation of the driving cycle is the main reason why the trade-off between speed and the other factors is observed as the more aggressive models perform well during the acceleration and worse when they have to brake unexpectedly. This is the case with the ACC which is a velocity matching controller, thus a part of the stimuli-response family of models, which are notorious for over-reacting to environmental changes and producing high acceleration/deceleration values.

C. *Analysis of Implications on Human and AV sub-populations*

Rapid acceleration adaptations and fast reaction times might enhance the performance of the AVs in the traffic system, however, we should also consider the effects on the performance of the human sub-population. Ideally, the presence of autonomous vehicles on the roads should also "steer" human drivers to also improve their performance indicators. Figure 3 shows the split of the improvement brought by the AVs between the two groups of vehicles. A split of 0.5 would mean that the performance increase on a given objective is perfectly shared by human drivers and AVs. A split higher than that would indicate that AVs are benefiting more than human drivers. It can be observed that the E-IDM provides better improvement splits for all objectives except efficiency where both splits are very close to 50% anyway. The biggest difference can be observed for the comfort objective. Higher magnitude of the split can be explained by the fact that the smoother driving style of the E-IDM model makes AVs act as a dampener between human drivers thus also improving their comfort levels.

VI. Conclusions, Limitations, and Future Work

This paper presented a comparative analysis of two longitudinal control algorithms, namely the ACC and the E-IDM car-following models, in a mixed traffic scenario of alternating human and AV string of vehicles. We have formulated 4 performance indicators of the presented traffic system: speed, efficiency, comfort, and safety. In order to find optimal parametrizations of the examined AV control models, we performed a multi-objective optimization and observed that the two models form a continuous Pareto front. A clear trade-off has been observed between the speed performance indicator and the rest of the indicators. The ACC resides within the higher velocity part of the front, while the E-IDM model covers the more comfortable efficient, and safe part of the front.

The results of the multi-objective calibration were used to scalarize the optimization problem and come up with a balanced single solution for optimal parameter sets of the two examined models. The E-IDM model exploits the trade-off by using a 2.5% reduction of speed to improve efficiency by 7 %, comfort by 50 % and arrive at a perfect safety score. It was further observed that the E-IDM model splits improvements of performance indicators more evenly between human drivers and autonomous vehicles.

The biggest limitation of our study is the human driver model. Even though HDM is considered to be reliable, the modelling of the perception errors produces unrealistically random fluctuations of the acceleration values for human-driven vehicles. This makes it extremely challenging for AVs to further improve traffic conditions even with perfect knowledge of the system, as predictions become obsolete. If a more adequate human driver model is present, a promising future direction of research should be the design of driving behaviour models that actively try to improve the driving style of the human drivers thus further enhancing the traffic system's performance.

Furthermore, the currently examined scenario is only a first step to tackle the problem of optimal AV behaviour in mixed traffic. The results of this initial experiment, therefore, must be validated and extended with more complex scenarios and include additional behavioural. Those include the consideration of multiple lanes and thus optimal lane changing models, random vehicle distribution on the road, variable penetration rate of autonomous vehicles, and urban road network scenarios including controlled and uncontrolled intersections. The evaluation of those more complex scenarios will extend the spectrum of traffic situations the models have to deal with and thus will produce more general results.

Last but not least, the imperfections of sensing and computational and communication delays of AVs should also be included in the respective models in order to only consider realistically applicable future AV control algorithms. It will therefore be interesting to see how well the imperfections of humans combine with the imperfections of AVs. The added complexity in: 1) the considered traffic scenarios, 2) the behavioural models of the vehicles, and 3) coordination efforts of the AVs, will set the stage for novel AV driving behaviour methods which optimize traffic on a system rather than vehicle level.

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